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Constraints on a General Higgs Sector from $K^0 - \bar{K}^0$, $B_d - \bar{B}_d$ Mixing and the ϵ Parameter

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Abstract

The scalar sector of the Standard Model is extended to include an arbitrary assortment of scalars. In the case where this assignment does not preserve $\rho = 1$ at the tree-level, the departure from unity itself puts the most stringent constraint on the scalar sector, and where $\rho_{tree} = 1$ is maintained, useful bounds on the parameter space of the charged Higgs mass and the doublet-nondoublet mixing angle can arise from data on $B_d - \bar{B}_d$, $K^0 - \bar{K}^0$ mixing and the ϵ parameter. These constraints turn out to be comparable (and in some cases, better) to those obtained from $Z \rightarrow b\bar{b}$ data.

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The electroweak symmetry breaking sector of the Standard Model (SM) is still as cloudy as it was in the time of its formulation; and the main factor responsible for this is the absence of any direct evidence of the Higgs boson. The minimal version of the SM requires one complex scalar doublet to break the electroweak symmetry; however, there is no a priori reason why more scalars cannot exist. Models with two or more doublets have been explored in this spirit [1].

It is also pertinent to investigate the consequences of scalars belonging to non-doublet representations of $SU(2)$. This will enlarge the particle content of the SM, and change the gauge-scalar as well as the fermion-scalar interactions, without affecting the $SU(2)_L \times U(1)_Y$ gauge structure of the model. That these non-doublet scalar representations can induce Majorana masses for left-handed neutrinos has been shown [2]. Collider signatures of scalars belonging to a triplet representation have also been investigated [3].

However, there is one serious constraint on these higher dimensional (> 2) scalar representations: they in general do not maintain $\rho = 1$ at tree-level. Singlet and doublet representations do not suffer from this malady and that is why much work have been done on their phenomenological implications [4, 5]. For an arbitrary assortment of scalars, one has three possibilities:

1. The higher dimensional multiplet does not *incidentally* contribute to ρ . This will happen, e.g., for a multiplet with weak isospin $T = 3$ and weak hypercharge $Y = 4$. However, being quite artificial, such representations will not be discussed anymore in this paper.
2. The vacuum expectation values (VEV) of the higher representations are much smaller than the doublet VEV so that $\rho - 1$ is within experimental bound.
3. There is a remaining custodial $SU(2)$ symmetry among the higher representations. In this case, the effects of the ‘bad’ representations on $\rho - 1$ cancel out. For such a cancellation to remain valid even at one-loop level, one requires a fine-tuning; however, it has been shown [5] that the fine-tuning required is of the same order as one encounters in

the SM. Following this prescription, some serious model-building has been done in recent times [5, 6].

Recently, a general formulation to treat arbitrary representations of scalars was proposed [7]. Only the constraint coming from the tree-level absence of flavour-changing neutral currents (FCNC) was assumed there. In simple terms, this constraint means that either a single weak doublet Φ_1 couples with both $T_3 = +1/2$ and $T_3 = -1/2$ fermions, or one weak doublet Φ_1 couples with $T_3 = +1/2$ and another, Φ_2 , couples with $T_3 = -1/2$ type fermions. For simplicity, we have assumed that the same doublet couples with quarks and leptons.

It was shown in Ref. [7] that if the arbitrary assortment of multiplets do not keep $\rho = 1$ at tree-level, then the constraint coming from $\rho = 1$ is by far the strictest to limit the doublet-nondoublet mixing. (This mixing occurs because, in general, the weak and the mass basis of scalars are not identical and states in these two basis are related by some unitary matrices.) However, for those models which keep $\rho_{tree} = 1$ (either entirely consisting of doublets and singlets, or having compensating ‘bad’ representations — possibility (3) as listed above), a significant constraint on the parameter space of the singly-charged Higgs mass m_{H^\pm} and doublet-nondoublet mixing angle θ_H can be obtained from $Z \rightarrow b\bar{b}$ data.

In this paper we investigate what constraints on the abovementioned parameter space can be obtained from processes like $B_d - \bar{B}_d$ and $K^0 - \bar{K}^0$ mixing, and from the experimental value of the CP -violating ϵ parameter. Such a study was performed earlier for two-Higgs doublet models [8]. We intend to show that the constraints sometimes turn out to be better than those obtained with the $Z \rightarrow b\bar{b}$ data. As already pointed out by Grossman [4], other processes do not play a significant role in constraining the parameter space. We will show that for those models where the scalar sector contains nondoublet representations, conclusions can differ significantly from the ones drawn in the case of multi-Higgs doublet model. It may again be stressed that we give a general treatment which yields the well-known results for multi-Higgs doublet model at proper limit.

Before proceeding further, let us set our notations, which will mostly follow Ref. [7]. In the weak basis, the Higgs multiplets are denoted by Φ and the fields by ϕ . In the mass basis, we use H to denote the fields. H and ϕ are related via unitary matrices; for our purpose it is sufficient to show a pair of such relations:

$$\begin{aligned} H_i^+ &= \alpha_{ij}\phi_j^+; & \phi_i^+ &= \alpha_{ji}^* H_j^+; \\ H_i^- &= \alpha_{ij}^* \phi_j^-; & \phi_i^- &= \alpha_{ji} H_j^-. \end{aligned} \quad (1)$$

We also set $H_1^+ \equiv G^+$, which means

$$G^+ = \sum_k \alpha_{1k} \phi_k^+. \quad (2)$$

Keeping the quarks in the weak basis, the Yukawa couplings are given by

$$\bar{u}dH_i^+ : \frac{ig}{\sqrt{2}m_W} \frac{\alpha_{i1}}{\alpha_{11}} (m_u P_L - m_d P_R) \quad (3)$$

for the case where only Φ_1 gives mass to both u - and d -type quarks, and

$$\bar{u}dH_i^+ : \frac{ig}{\sqrt{2}m_W} \left(\frac{\alpha_{i1}}{\alpha_{11}} m_u P_L - \frac{\alpha_{i2}}{\alpha_{12}} m_d P_R \right) \quad (4)$$

for the case where Φ_1 (Φ_2) gives mass to u (d)-type quarks. The projection operators are

$$P_L = (1 - \gamma_5)/2, \quad P_R = (1 + \gamma_5)/2. \quad (5)$$

These two models will henceforth be called Model 1 and Model 2 respectively. Consideration of quarks in the mass basis will introduce the relevant elements of the quark mixing matrix.

In the SM, the short-distance part of Δm_K , the $K_L - K_S$ mass difference, is given by

$$\Delta m_K = \frac{G_F^2}{6\pi^2} \eta_1 m_K B_K f_K^2 |(V_{cd}^* V_{cs})|^2 m_c^2 I_1(x_c), \quad (6)$$

where η_1 takes care of the relevant short-distance QCD correction, and f_K is the kaon decay constant. B_K parametrizes the error in using vacuum insertion approximation to evaluate the matrix element $\langle \bar{K} | \bar{d}\gamma^\mu (1 - \gamma_5) s \bar{d}\gamma_\mu (1 -$

$\gamma_5)s|K\rangle$, and lies between 0 and 1. Using chiral perturbation theory as well as hadronic sum rules, one obtains $B_K = 1/3$ [9], whereas lattice QCD studies give $B_K = 0.85$ as the central value [10]. Other values like $B_K = 0.70$ is obtained from $1/N$ expansion technique [11].

The function $I_1(z)$ has the expression

$$I_1(z) = 1 - \frac{3z(1+z)}{4(1-z)^2} - \frac{3z^2 \ln(z)}{2(1-z)^3}, \quad (7)$$

and for any quark q , we use $x_q = m_q^2/m_W^2$.

Parametrizing the quark mixing matrix in an approximate form [12]

$$V_{CKM} = \begin{pmatrix} 1 & s_{12} & s_{13}e^{-i\delta} \\ -s_{12} - s_{13}s_{23}e^{i\delta} & 1 & s_{23} \\ s_{12}s_{23} - s_{13}e^{i\delta} & -s_{23} & 1 \end{pmatrix}, \quad (8)$$

where the cosines of the mixing angles have been approximated by unity and s_{13} is assumed to be one order of magnitude smaller than s_{12} and s_{23} , one obtains

$$|(V_{cd}^* V_{cs})|^2 = s_{12}^2 + s_{23}^4 q^2 + 2s_{12}s_{23}^2 q \cos \delta \quad (9)$$

with $q = s_{13}/s_{23}$.

Expression for the $B_d - \bar{B}_d$ mixing parameter, x_d , in the SM, is [13]

$$x_d = \frac{\Delta m}{\Gamma} \bigg|_{B_d} = \tau_b \frac{G_F^2}{6\pi^2} \eta_B B_B f_B^2 m_B m_t^2 I_1(x_t) |V_{td}^* V_{tb}|^2 \quad (10)$$

where η_B is the corresponding short-distance QCD correction. $\sqrt{B_B} f_B$ is estimated from the lattice studies to be 0.14 ± 0.04 GeV. $\tau_b |V_{td}^* V_{tb}|^2$ can be written as

$$\tau_b |V_{td}^* V_{tb}|^2 = \tau_b |V_{cb}|^2 (s_{12}^2 + q^2 - 2s_{12}q \cos \delta). \quad (11)$$

Lastly, the CP -violating parameter of the neutral kaon system, ϵ , has the following expression in the SM:

$$|\epsilon| = \frac{G_F^2}{12\pi^2} \frac{m_K}{\sqrt{2}\Delta m_K} m_W^2 B_K f_K^2 \left[\eta_1 \text{Im}(V_{cd}^* V_{cs})^2 x_c I_1(x_c) \right. \\ \left. + \eta_2 \text{Im}(V_{td}^* V_{ts})^2 x_t I_1(x_t) + 2\eta_3 \text{Im}(V_{cd}^* V_{cs} V_{td}^* V_{ts}) x_c I_2(x_c, x_t) \right], \quad (12)$$

where, apart from the symbols previously explained,

$$I_2(z_1, z_2) = \ln(z_2/z_1) - \frac{3}{4} \frac{z_2}{1-z_2} \left[1 + \frac{z_2}{1-z_2} \ln z_2 \right]. \quad (13)$$

η_1 , η_2 and η_3 are three QCD correction factors, η_1 being the same as in eq. (6).

Now let us concentrate on the contributions to the abovementioned parameters coming from an extended Higgs sector. Our discussion will be limited within those assortment of scalar multiplets which keep $\rho_{tree} = 1$; however, a generalization is straightforward but of little physical importance.

One has to consider two new box diagrams: one with two charged Higgses and two up-type quarks, and one with one charged Higgs, one W^+ and two up-type quarks. Note that as per eq. (2), the new physics contribution should exclude the diagrams containing only H_1^+ and no other charged Higgses.

To avoid cumbersome formulae which do not shed much light to new physics issues, we assume all charged Higgses to be degenerate in mass [7]. This is not a too drastic approximation if one considers the fact that it is the mass of the charged Higgs, m_{H^+} , which we want to constrain. In case the charged Higgses do not have the same mass, m_{H^+} corresponds to the lightest physical one. To do meaningful numerology, one has either to assume that all H^+ s are degenerate, or that one of them is light enough to contribute and the others are so heavy that they effectively decouple. However, physically interesting models [6] do have all scalar masses of the same order of magnitude, and so we stick to the first approximation. It may be mentioned that if the masses of the charged scalars are not exactly the same but similar in magnitude, bounds that we obtain change very little. We will also state what happens if one considers the second limit, i.e., existence of only one ‘light’ charged scalar.

Another reasonable approximation is to take all other quarks except the top to be massless while considering their couplings to the scalar fields. This makes eqs. (3) and (4) identical, and the results thus obtained will be more general. Note that as the scalar coupling to fermion-antifermion pair is proportional to the fermion mass, the GIM mechanism is not operative.

We give expressions for the contributions of the scalar-mediated diagrams to Δm_K , x_d and ϵ . For any general quark q , we use $y_q = m_q^2/m_{H^+}^2$. As all m_{H^+} s are assumed to be same, y_q is unique.

The contribution to Δm_K is

$$\Delta m_K^H = \frac{G_F^2}{24\pi^2} \eta_1 m_K B_K f_K^2 |V_{td}^* V_{ts}|^2 m_t^2 (J_{HH} + J_{HW}) \quad (14)$$

where

$$J_{HH} = \sum'_{i,j} y_t \frac{\alpha_{i1}^2 \alpha_{j1}^2}{\alpha_{11}^4} \left[\frac{1+y_t}{(1-y_t)^2} + \frac{2y_t \ln y_t}{(1-y_t)^3} \right], \quad (15)$$

and

$$J_{HW} = \sum_{i=2}^n x_t \left(\frac{\alpha_{i1}^2}{\alpha_{11}^2} \right) [2I_3(x_t, x_H) - 8I_4(x_t, x_H)]. \quad (16)$$

Here \sum' means that the sum over both the mass indices runs from 1 to n , the number of charged scalars, but $i=1, j=1$ term corresponding to the Goldstone contribution is to be subtracted, as that is already considered in the SM amplitude. The same logic applies for the sum in eq. (16). The expression for the two functions, I_3 and I_4 , are given by

$$I_3(x_H, x_t) = \frac{x_t}{(1-x_t)(x_H-x_t)} - \frac{x_H^2 \ln x_H}{(1-x_H)(x_t-x_H)^2} + \frac{x_t(2x_H-x_t-x_t x_H) \ln x_t}{(x_H-x_t)^2(1-x_t)^2}, \quad (17)$$

$$I_4(x_H, x_t) = -\frac{1}{(1-x_t)(x_H-x_t)} + \frac{x_H \ln x_H}{(1-x_H)(x_t-x_H)^2} - \frac{(x_H-x_t^2) \ln x_t}{(x_H-x_t)^2(1-x_t)^2}, \quad (18)$$

with $x_H = m_{H^+}^2/m_W^2$. Note that I_3 differs in sign from that given in eq. (B.3) of Ref. [4].

x_d is enhanced by

$$x_d^H = \tau_b \frac{G_F^2}{24\pi^2} \eta_B B_B f_B^2 m_B m_t^2 |V_{td}^* V_{tb}|^2 (J_{HH} + J_{HW}), \quad (19)$$

and the contribution to ϵ is

$$|\epsilon|^H = \frac{G_F^2}{48\pi^2} \frac{m_K}{\sqrt{2}\Delta m_K} m_W^2 B_K f_K^2 m_K \eta_2 x_t \text{Im}(V_{td}^* V_{ts})^2 (J_{HH} + J_{HW}). \quad (20)$$

None of the above charged scalar mediated processes are possible if $\alpha_{i1} = 0$ for $i \neq 1$. In other words, the charged scalar of the weak doublet that gives mass to the top quark must mix with charged scalars of other multiplets to produce such contributions. This mixing is parametrised by θ_H , i.e., $\alpha_{i1} = \cos \theta_H$. From the unitarity of the α matrix, $\sum_{i=2}^n |\alpha_{i1}|^2 = \sin^2 \theta_H$.

Thus, if all H^+ s are degenerate, J_{HH} is proportional to $\sec^4 \theta_H - 1$ and J_{HW} is proportional to $\sec^2 \theta_H - 1$. However, if only the k -th charged scalar effectively contributes, the element $|\alpha_{k1}|^2$, and not the sum, gets paramount importance. It may happen that $|\alpha_{k1}|^2$ is very small or actually zero. Such a thing happens if H_5^+ is the lightest charged scalar in the triplet model of Ref. [6]. In this case, all our discussions are invalidated, and we arrive at the well-known result of possible existence of a light charged scalar which does not couple to fermions.

Assuming the degeneracy of m_{H^+} , we try to put constraints on $m_{H^+} - \tan \theta_H$ plane. A major obstacle in that direction is the fact that a lot of quantities like B_K , $B_B f_B^2$, s_{23} , δ , and even m_t , are poorly known or estimated. To be consistent with the present experimental data, we take [14, 15]

$$\begin{aligned} G_F &= 1.16639 \times 10^{-5} \text{ GeV}^{-2}, \quad \Delta m_K = 3.52 \times 10^{-15} \text{ GeV}, \\ m_{B_d} &= 5.28 \text{ GeV}, \quad m_t = 176 \text{ GeV}, \quad m_W = 80.41 \text{ GeV}, \\ x_d &= 0.77, \quad |\epsilon| = 2.26 \times 10^{-3}, \quad f_K = 0.165 \text{ GeV}, \quad m_K = 0.498 \text{ GeV}, \\ s_{12} &= 0.2205, \quad s_{23} = 0.040, \quad \tau_b |V_{cb}|^2 = 3.5 \times 10^{-9} \text{ GeV}^{-1}. \end{aligned} \quad (21)$$

The numerical values of the QCD correction factors that we use are [16, 17]

$$\eta_1 = 0.78, \quad \eta_2 = 0.60, \quad \eta_3 = 0.37, \quad \eta_B = 0.85. \quad (22)$$

First, let us concentrate on Δm_K . Assuming no long-distance contribution, Δm_K does not limit $\tan \theta_H$ significantly. For $B_K = 1/3$, the maximum value of $\tan \theta_H$ is 7.4, 6.2 and 7.6 for $m_{H^+} = 100, 200$ and 500 GeV respectively. This bound is one order of magnitude poorer than that derived from

$Z \rightarrow b\bar{b}$ data. Though formally eqs. (6) and (10) contain δ , the result is insensitive to its specific value; the reason is the small coefficient of $\cos\delta$ in eq. (9). For $B_K = 0.85$, the bounds are a shed better: $\tan\theta_H(\max) = 3.5$, 2.9 and 3.6 for $m_{H^+} = 100$, 200 and 500 GeV. We note that the bound is ‘strongest’ at around $m_{H^+} = m_t$.

The situation is different if one has, say, a 50% long-distance contribution. In that case, $B_K = 1/3$ gives $\tan\theta_H(\max) = 4.5$, 3.7 and 4.6 for $m_{H^+} = 100$, 200 and 500 GeV. However, $B_K = 0.85$ oversaturates the SM value and no room for new physics is left. $B_K = 0.70$ yields a fairly strong constraint: $\tan\theta_H(\max) = 1.2$, 1.0 and 1.2 for the three values of m_{H^+} we have chosen to mention. This is comparable to those bounds obtained from partial width of Z into $b\bar{b}$ pairs.

With $B_K = 1/3$, $s_{23} = 0.040$ and $q = 0.10$, the strongest bound on $\tan\theta_H(\max)$ is 1.1, which is for $\delta = 7\pi/12$ and $m_{H^+} = 300$ GeV. For $q = 0.06$, the bound is somewhat less stringent; the results are shown in Figs. (1a) and (1b). Also, $q = 0.14$ constrains the parameter space more tightly. Furthermore, one observes that $B_K = 0.85$ does not allow $\delta > \pi/4$, and $B_K = 0.70$ does not allow $\pi/4 < \delta < 3\pi/4$ — the SM value saturates the experimental number. Even for those values of δ which allows for a new physics contribution, $\tan\theta_H(\max)$ is generally less than 1.0, which is a better constraint than that obtained from $Z \rightarrow b\bar{b}$ data.

Currently favoured values of $B_B f_B^2$ (≈ 0.02 GeV²) also does not allow $\delta > \pi/2$ from measurements on x_d . For $\delta = \pi/2$, one gets $\tan\theta_H(\max) = 0.36$ for $m_{H^+} = 100$ GeV. Even for δ as low as $\pi/6$, $\tan\theta_H(\max) = 1.4$ for a charged scalar mass of 100 GeV. Lowering $B_B f_B^2$ to 0.01 GeV² results in a larger allowed value of $\tan\theta_H(\max)$. Figs. (2a) and (2b) show the detailed result.

We conclude from this analysis that even in a model with arbitrary assortment of scalars, one can obtain fairly strong constraints on the parameter space of the scalar sector, with a very few reasonable assumptions, from $B_d - \bar{B}_d$ mixing data and the ϵ parameter, and maybe even from $K^0 - \bar{K}^0$ mixing data. These constraints are shown to be comparable, and sometimes better, to those obtained from $\Gamma(Z \rightarrow b\bar{b})$, which was calculated in Ref. [7].

We want to remind our readers that such an analysis is only meaningful if the lightest physical charged scalar(s) couple with fermions, and if $\rho_{tree} = 1$ is maintained (otherwise, ρ parameter puts a better constraint). The error bar in m_t turns out to be insignificant; however, quantities like B_K , $B_B f_B^2$ and δ , which are either poorly known or completely unknown, play a significant role. With a more accurate experimental determination of these quantities, one hopes to make these constraints more meaningful.

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Figure Captions

1(a). Upper limits on $\tan\theta_H$ for different values of m_{H^+} , as obtained from the analysis of the ϵ parameter. We take $q = 0.10$. The uppermost curve is for $\delta = \pi/6$, and the successive ones are for $\delta = \pi/4, \pi/3, 5\pi/12, 3\pi/4$ and $7\pi/12$ respectively.

1(b). Same as in 1(a), with $q = 0.06$. The curves are for $\delta = \pi/6, \pi/4, \pi/3, 3\pi/4, 5\pi/12$ and $7\pi/12$ respectively.

2(a). Upper limits on $\tan\theta_H$ for different values of m_{H^+} , as obtained from the analysis of $B_d - \bar{B}_d$ mixing. We take $B_B f_B^2 = 0.02$. The uppermost curve is for $\delta = \pi/6$, and the successive ones are for $\delta = \pi/4, \pi/3, 5\pi/12$ and $\pi/2$ respectively. For $\delta > \pi/2$, the SM value saturates the experimental bound.

2(b). Same as in 2(a), with $B_B f_B^2 = 0.01$. The curves are for $\delta = \pi/6, \pi/4, \pi/3, 5\pi/12, \pi/2, 7\pi/12, 2\pi/3, 3\pi/4$ and $5\pi/6$ respectively.

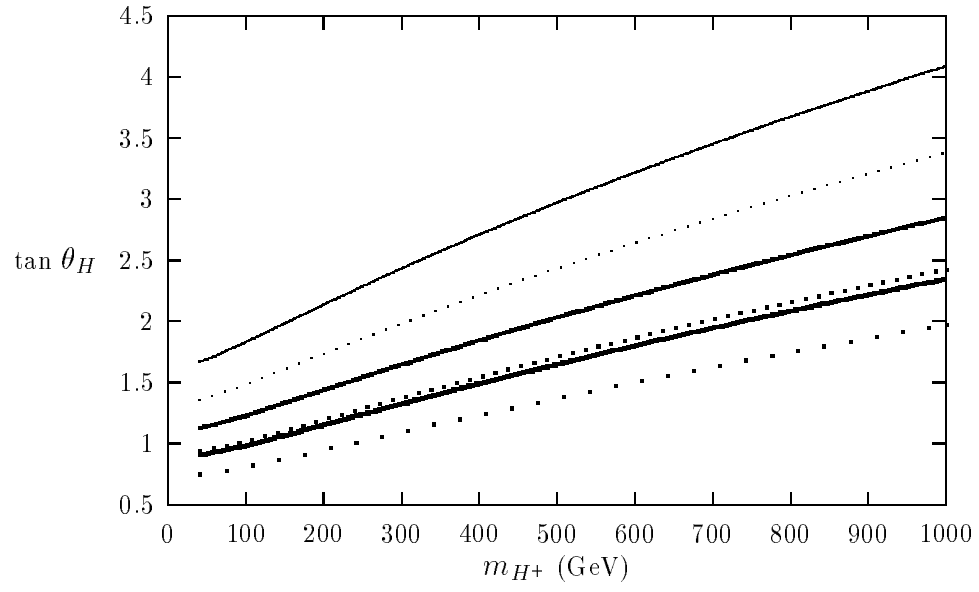


Fig. 1(a)

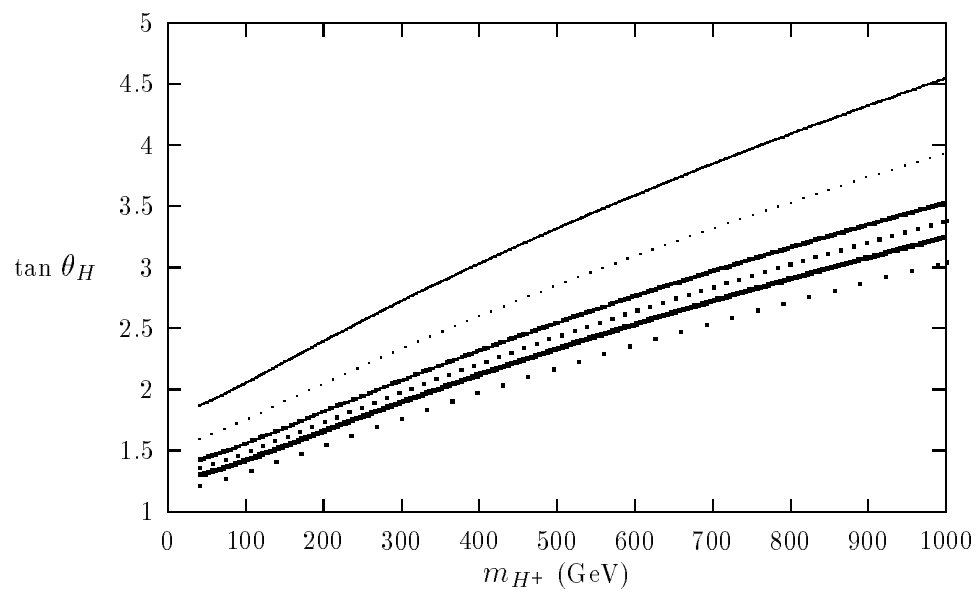


Fig. 1(b)

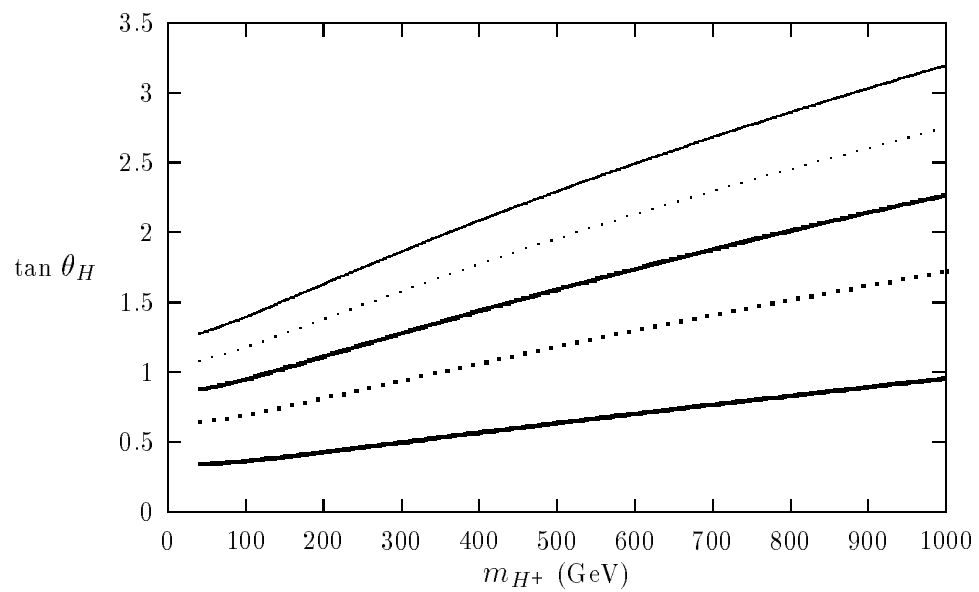


Fig. 2(a)

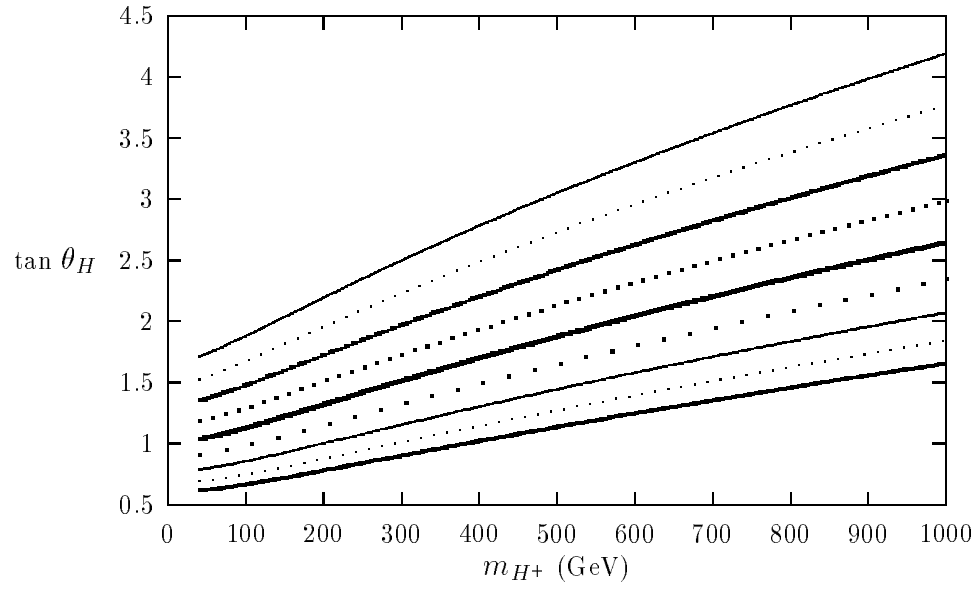


Fig. 2(b)